Quick instruction of "Brownian Motion Online Observation" and Some Historical Comments

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Brownian motion is random motion of small particles, which are called Brownian particles, in fluid such as water. A number of molecules composing the fluid randomly hit an particle due to their thermal motion. The effect of the impacts of the molecules to the Brownian particle is incompletely averaged when the size of the Brownian particle is small enough (~1 μm). Then the fluctuation of the motion of the molecules is observed as the Brownian motion.

You can observe the Brownian motion by this online software. The average diameter of the Brownian particles (Polybead Microspheres, Polysciences) is 0.51 μm. The fluid is water, and the observation was carried out at 26 °C = 299 K using a kind of microscope (Digital-microscope VH-5500, Keyence). The image is magnified by 2000 times and one mesh corresponds to 5 μm.

The video of the Brownian motion starts by clicking the "video start/stop" button. Now set your eyes on a particle around the center of the image. Next you click the "timer start/stop" button to start the count down from -10 second. You need to chase your particle by the mouse cursor and click the mouse on the particle when time becomes 0. You repeat the same thing every 30 seconds till time becomes 300 seconds. The positions of the particle appear in the bottom table.

You will sometimes lose the particle when it escapes from the focus due the vertical motion. And the particle may go out of the image area. When you lose the particle before time becomes 300 seconds, you need to stop the timer by pressing the "timer start/stop" button again and start over the observation from the beginning.

Why you need to record the position of the particle?

By collecting the 10 data sets on 10 particles, you can estimate Avogadro constant N_A by utilizing Einsteins's relation,

$$D = \frac{RT}{6\pi N_A \eta a}$$

where D is the diffusion coefficient, R is the gas constant, T is the absolute temperature, η is the viscosity of water, a is the radius of the Brownian particle, respectively. You can determine the D from the following relation.

$$\sigma^2 = 4Dt$$

The σ is the standard deviation of the positions of a group of Brownian particles at the time t in the unit of second. This relation says that the variance ($= \sigma^2$) is proportional to time and the coefficient is $4D$ in case of the two-dimensional motion.

You can convert a set of data $\{x, y\}$ in the table to $\{\Delta x, \Delta y\}$, namely, the displacement of the particle from the position at $t = 0$. After you finish collecting the 10 data sets, you can calculate the σ^2 at each time (t = 30 s, 60 s, ⋯, 300 s) as follows.

$$\sigma^2(t) = \frac{\sum_{i=1}^{10}\left(\left(\Delta x(t)\right)^2 + \left(\Delta y(t)\right)^2\right)}{9}$$

The reason why the denominator is not 10 but 9 is that the σ is now so-called "sample standard deviation". Please refer to textbooks of statistics for the difference between the "sample standard deviation" and "population standard deviation". Also note that we assumed here the mean of the displacement from the origin to be zero, though it holds only when the number of particles is large enough. Anyway, now you can calculate the $4D$ as a slope of the $\sigma^2(t)$ vs. t. Since the number of particles is just 10, the plotted σ^2 against t will be rather scattered. So here we attempt to determine the slope $4D$ by the least-squares fit to a line to the data points. On the other hand, you have the value of a as 0.51 μm / 2. The η of the water at T = 299 K and R are also available somewhere. When you make a math accurately, you will obtain a value N rather close to the known value $N_A = 6.02 \times 10^{23}$ mol^{-1}. Do not expect an exact value. It is sufficient when you have a value with the same order of N_{A} since you chased only 10 particles for 5 minutes each!

This is the very simplified version of the similar but much more thorough experiments carried out by J. B. Perrin reported in 1908. He has succeeded to prove the first direct evidence of existence of atoms and molecules by utilizing Einstein's relation shown above. Einstein theoretically derived the relation without knowing the Brownian motion but by assuming the existence of fluid molecules. Perrin was honored with the Nobel Prize of Physics in 1926 for this work.

The simplified experiment shown here was originally designed as a theme of the student experiment course by Emeritus Professor Hiroumi Ishii in 1996 at Osaka City University. I took it from him in 1997 and since then I have been providing and improving the experiment. In 2020 we had to provide a part of the student experiment online due to the COVID-19 outbreak. That is the reason why I created the online version of the experiment. I am very happy if it is also helpful to students out of our university in a similar situation.